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A CEREAL CROP CANOPY LIGHT DISTRIBUTION AND PHOTOSYNTHESIS MODEL BASED ON MULTIPLE FACTORS – MODELING AND SIMULATION

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Abstract

Canopy light distribution and photosynthesis modeling is fundamental to cereal crop cultivation, breeding and crop informatics. It also has a great theoretical and practical significance for the evaluation and optimization of plant types and computer simulations of crop growth. This study has developed a cereal crop canopy photosynthesis model based on the improved "stratified-clipping method", which combines morphology, physiology and optics. This model includes a canopy shape model, a single leaf photosynthesis rate model, a canopy light distribution model and a photosynthetic rate model. In this study we carried out a numerical simulation of the photosynthetic rates of the 15625 rice plant types. The numerical results showed that the photosynthesis rate was closely related to the following five factors: leaf density, leaf nitrogen content, leaf length, leaf width and leaf angle. The model led us to the conclusion that the ideal rice plant type has large values around the vectors for the five factors in the upper part of the canopy, but should decreases downwardly along the canopy

Introduction

The radiation flux vertical gradient in the plant canopy is a measure of the energy absorbed by leaves at different heights. The radiation distribution is believed to be the most important factor within the micro-climates of plant communities (Monteith, 1965; Monsi & Saeki, 1980 and 2005) developed Jensen's study (Boysen, 1932) and published the first significant mathematical model for canopy photosynthesis. Based on the Monsi-Saeki model, many researchers have studied plant canopy photosynthesis in theory and prictice, for example, Loomis & Williams, 1963; Ross, 1981; de Wit, 1965; Yu *et al.*, 1998a, 1998b. The "stratified-clipping method", as the analytical method for establishing the Monsi-Saeki model, has been widely applied in canopy photosynthesis research. Hirose (2005) stated that canopy photosynthesis model studies are important in quantifying the biochemical function of chloroplasts, canopy carbon dioxide fixation, analyzing resource usage strategies under different light and nitrogen conditions by species and individual plants and predicting the future structure of terrestrial plant communities. The groundbreaking significance of the Monsi-Saeki model on plant photosynthesis was confirmed by later studies. However, Hirose (2005) showed that not every leaf photosynthetic characteristic was the same in the Monsi-Saeki model. For example, the leaf nitrogen content, a vital element of the leaf photosynthesis rate, varied along the base position (Farquhar, 1989; Jan, 2011; Hammad, 2011; Hirose, 1971; Hirose & Werger, 1987a, 1987b, 1994, 1995; Hirose *et al*., 1988, 1989; Iwaki *et al*., 1969; Kitamoto, 1972; Moosavi, 2012). Farquhar (1989) suggested that canopy photosynthesis should have been maximized theoretically when the irradiance received by each leaf was proportional to its nitrogen content. Kuroiwa (1980) stated that leaf angle effects on canopy photosynthesis also varied along base positions. Monsi had to make assumptions because the "stratified-clipping method" used in the Monsi-Saeki model cannot distinguish between the different base position leaves in a layer. The "stratified-clipping method" also has other drawbacks, such as destruction to the plant canopy, which prevents continued observation.

Based on the characteristics of the cereal crop canopy, this study undertook further development of the "stratified-clipping method", and created a light distribution and photosynthesis model. Many factors are included in this model, such as leaf density, leaf nitrogen content, leaf length, leaf width, leaf angle and leaf azimuth distribution, which vary along the base position and are closely related to solar radiation distribution in the canopy and the photosynthesis rate. Finally, using this model, numerical simulation of the photosynthetic rate of the proposed 15625 rice plant types was undertaken and the simulation results were analyzed in detail.

Study methods and data processing: We framed the Descartes coordinates as follows: the z coordinate is a vertical direction with a positive downward direction; the x coordinate is in the west-east direction and y is the north-south direction, A XOY plane represents the canopy upper level. Some parameters are defined and noted as follows: leaf base position refers to the z coordinate of the leaf base, denoted by z; leaf length refers to the length of the leaf vein, denoted by a; leaf width refers to the maximum width of the leaf, denoted by b; leaf inclination angle refers to the angle between the vein and the horizontal plane, denoted by α and leaf azimuth angle refers to the angle between the leaf vein's projection in the horizontal plane and the x coordinate, denoted by β. The data were measured by random sampling and are shown in Table 1.

$z_{\max}$, z_{min}□respectively denote the leaf base position of the highest and the lowest leaf and Ln is the number of leaves within a square meter. The interval, $[z_{\max}, z_{min}]$, was divided into m sub- equal intervals, ith is denoted by $[Iz_i, Iz_{i+1}]$, in which the mean length was $\bar{a}_i$, the mean width was $\overline{b}_i$, the mean inclination angle was □ and the mean nitrogen content was $\overline{N}_i$. Similarly, a circle $[o, 2\pi]$ was divided into l equal sub-intervals and the ith sub-interval was denoted by [$I\beta_i$, $I\beta_{i+1}$], in which there were l_i leaf azimuths.

Table 1. Abbreviations used in the data modeling.

Leaf No.	Shading coefficient	Base position	Leaf length	Leaf width	Inclination angle	Azimuth angle	Nitrogen level
i	K_i	z_i	a_i	b_i	α_i	β_i	N_i

Modeling

Canopy shape modeling: We assumed that leaf nitrogen content, leaf density, leaf length, leaf width and leaf angle were functions of the leaf base positions. We also assumed that all of the leaves had an identical shape. Linear/nonlinear regression analyses were used for leaf density, which could be expressed as the ratio of the leaf number within a unit stem to the total leaf number using the equation:

$$h(z)=\begin{cases}\dfrac{m_i}{Ln} & Iz_i \le z < Iz_{i+1}\\ 0 & z_{\max} \ge z, or, z < z_{\min}\end{cases},$$

where $\int_{h_{\min}}^{h_{\max}} h(z)dz = 1$. This was fitted as a continuous function and was denoted as *h(z)*. Similarly, we built in the leaf nitrogen content function, *N(z)*, the leaf length function, *a(z)*, the leaf width function, *b(z)*, the leaf inclination angle function, *α(z)* and the leaf azimuth function, *τ(β)*, where *N(z)*, *a(z)*, *b(z)*, *α(z)* and *τ(β)* were all continuous functions and *τ(β)* was the 2π periodic function and $\int_0^{2\pi} \tau(\beta)d\beta = 1$.

We also framed the Descartes coordination. The leaf base position was the origin of coordinate and the leaf vein was the x coordinate. The leaf shape function, which described the leaf boundary in the first phase, according to the symmetry of leaves on the veins, was denoted by $Vb(x,a,b)$. By integration, we can get the formula for a single leaf area as follows:

$$S(a,b) = 2\int_0^a Vb(x,a,b)dx.$$

By combining the leaf shape function, $Vb(x,a,b)$ and the leaf density function, we got the leaf area density function, which could be expressed as the leaf area within a unit stem at z, as follows:

$$s(z) = \int_z^{+\infty} Ln \bullet h(v) \bullet \frac{Vb\left(\dfrac{v-z}{\cos(\alpha(v))}\right)}{\cos(\alpha(v))}dv.$$

Similarly, we got the leaf area accumulated function, which could be expressed as the leaf area within a unit stem from 0 to z, as follows:

$$S(z) = \int_0^z s(u)du = \int_0^z \int_u^{+\infty} Ln \bullet h(v) \bullet \frac{Vb\left(\dfrac{v-u}{\cos(\alpha(v))}\right)}{\cos(\alpha(v)}dvdu$$

,
and the LAI could be expressed as follows:

$$LAI = \int_{z_{\min}}^{z_{\max}} Ln \bullet h(z) \bullet S(a(h(z)), b(h(z)))dz.$$

Leaf photosynthetic rate model: Hirose & Werger (1987a, 1987b) stated that the leaf net photosynthesis function for photosynthetic photon flux density (PPFD) and leaf nitrogen density could be described by a non-rectangular hyperbolic equation:

$$p(I,N) = \frac{\phi I + p_{\max} - \sqrt{(\phi I + p_{\max})^2 - 4\theta\phi I p_{\max}}}{2\theta} - r$$

,
where: ϕ is the initial slope; $p_{\max}$ is the light-saturated rate of photosynthesis; θ and is a curvature factor and r is dark respiration. Hirose & Werger (1987a, 1987b) suggested that these parameters were a function of leaf nitrogen density. Fig. 1 shows the photosynthetic rates response to light intensity under different nitrogen contents.

Solar Radiation Model: When the order of the day is Dn, the astronomy irradiance formula at time T is:

$$Q_0(T) = I_0 E_0 (\sin\psi \sin\delta + \cos\psi \cos\delta \cos\varpi),$$

where I_0 is the solar constant, E_0 is the correction coefficient for the distance from the earth to the sun, ψ is the latitude , δ is solar elevation angle (which is the function of the order of the day when δ(θ) = 0.006894-0.399512cos(θ)+0.07205 sin(θ)-0.006799cos(2θ) + 0.00089sin(2θ) – 0.002689cos(3θ) – 0.001516 sin(3θ), where $\theta = 2\pi(Dn-1)/365$) and ϖ is the sun azimuth (which is the function of T and θ $\varpi(T) = \left(\varpi_a + 15\pi(T - T_a)/180\right)$, where $\varpi_a = -\arccos(-\tan\psi \tan\delta)$ and $Ta = 12 + \varpi_a \times 180/15\pi$) (Zuo, 1991). With the exception of cloudy weather, there was little change in the clear index, so the direct radiation and diffuse radiation could be expressed by $D(T) = C_D Q_0(T)$ and $S(T) = C_s Q_0(T)$, respectively, where C_D is the clear index of direct radiation and C_s is the clear index of diffuse radiation.

Canopy photosynthesis model

Direct radiation model: If the sun angle is sh (which can be calculated using the formula: $sh(\psi,\delta,\varpi) = \arcsin(\sin\psi \sin\delta + \cos\psi \cos\delta \cos\varpi)$), then the radiation direction is parallel to coordinate Y and the projected area, A, of the leaves with inclination angle, α, azimuth, β, area, S, can be computed as follows:

when $\beta \in [0,\pi]$, then:

$$A(\alpha,\beta,sh) = S\times(\cos\alpha + \sin\alpha \tan sh \sin\beta)$$

When $\beta \in [\pi,2\pi]$ and $\sin(\beta-\pi) \le \cot\alpha \cot\theta$ then

$$A(\alpha,\beta,sh) = S\times(\cos\alpha - \sin\alpha \tan sh \sin(\beta-\pi))$$

and

when $\beta \in [\pi,2\pi]$ and $\sin(\beta-\pi) \le \cot\alpha \cot\theta$ then:

$$A(\alpha,\beta,sh) = S\times(-\cos\alpha + \sin\alpha \tan sh \sin(\beta-\pi))$$

.

Furthermore, when the order of the day is Dn, the time is T, the latitude is ψ, the sun azimuth is $\varpi(T)$, the daily angle is $\theta = 2\pi(Dn-1)/365$, solar declination is $\delta(\theta)$ and the solar altitude angle is $sh(\delta,\psi,\varpi(T))$, then the mean projective area on the horizon of a unit area leaf with inclination angle, α, azimuth, β and an object to distribution, $\tau(\beta)$, can be computed by

$$EA(\alpha,T,D_n) = \int_0^{2\pi} \tau(\beta-\varpi(T))A(\alpha,\beta,sh))d\beta .$$

If $sh > \alpha$, then:

$$EA(\alpha,T,D_n) = \int_0^{\pi} \tau(\beta-\varpi(T))(\cos\alpha + \sin\alpha \tan sh \sin\beta))d\beta + \int_{\pi}^{2\pi} \tau(\beta-\varpi(T))(\cos\alpha - \sin\alpha \tan sh \sin\beta))d\beta$$

If $sh \le \alpha$, then:

$$\begin{aligned} EA(\alpha,T,D_n) = &\int_0^{\pi} \tau(\beta-\varpi(T))(\cos\alpha + \sin\alpha \tan sh \sin\beta))d\beta \\ &+ \int_{\pi}^{\pi+\arcsin(\cot\alpha\cot sh)} \tau(\beta-\varpi(T))(\cos\alpha - \sin\alpha \tan sh \sin(\beta-\pi))d\beta \\ &+ \int_{\pi+\arcsin(\cot\alpha\cot sh)}^{2\pi-\arcsin(\cot\alpha\cot sh)} \tau(\beta-\varpi(T))(-\cos\alpha + \sin\alpha \tan sh \sin(\beta-\pi))d\beta \\ &+ \int_{2\pi-\arcsin(\cot\alpha\cot sh)}^{2\pi} \tau(\beta-\varpi(T))(\cos\alpha - \sin\alpha \tan sh \sin(\beta-\pi))d\beta \end{aligned}$$

Intercepting a layer from the position z, ($z+\Delta z$) and when the radiation intensity on the upper canopy layer, which is intercepting from position z and $z+\Delta z$, is $I(z)$, then the lower radiation intensity $I(z+\Delta z)$ can be approximately computed by:

$$I(z+\Delta z) = I(z)\left(1-\overline{s}(z,\Delta z)\right),$$

where
$$\overline{s}(z,\Delta z,T,D_n) = \int_z^{+\infty} Ln \bullet h(v) \bullet \frac{Vb\left(\dfrac{v-z}{\cos(\alpha(v))}\right)\bullet \Delta z}{\cos(\alpha(v))} \bullet EA\left(\alpha(v),\varpi(T),sh\right)dv .$$

If you divide interval $[0,z]$ into $\hat{n}$ sub-intervals, then the radiation intensity at position z is:

$$I(z) = I(0)\left(1-\overline{s}(0,\frac{z}{\hat{n}},\varpi(T),sh)\right)\bullet\left(1-\overline{s}(\frac{z}{\hat{n}},\frac{2z}{\hat{n}},\varpi(T),sh)\right)\bullet\cdots \bullet\left(1-\overline{s}(\frac{z}{\hat{n}},\frac{(\hat{n}-1)z}{\hat{n}},\varpi(T),sh)\right).$$

Taking a logarithm on both sides, we got:

$$\ln I(z) = \ln I(0) + \ln\left(1-\overline{s}(0,\frac{z}{\hat{n}},\varpi(T),sh)\right) + \ln\left(1-\overline{s}(\frac{z}{\hat{n}},\frac{2z}{\hat{n}},\varpi(T),sh)\right) + \cdots + \ln\left(1-\overline{s}(\frac{z}{\hat{n}},\frac{(\hat{n}-1)z}{\hat{n}},\varpi(T),sh)\right).$$

when $\hat{n}$ is large enough, then:

$$\ln\left(1-\overline{s}(\frac{z}{\hat{n}},\frac{kz}{\hat{n}},\varpi(T),sh)\right)\approx-\overline{s}(\frac{z}{\hat{n}},\frac{kz}{\hat{n}},\varpi(T),sh)$$

So:

$$\ln I(z)\approx\ln I(0)-\overline{s}(0,\frac{z}{\hat{n}},\varpi(T),sh)-\overline{s}(\frac{z}{\hat{n}},\frac{2z}{\hat{n}},\varpi(T),sh)-\cdots-\overline{s}(\frac{z}{\hat{n}},\frac{(\hat{n}-1)z}{\hat{n}},\varpi(T),sh)$$

$$=\ln I(0)-\int_0^z\int_z^{+\infty}Ln\bullet h(v)\bullet\frac{Vb\left(\frac{v-z}{\cos(\alpha(v))}\right)}{\cos(\alpha(v))}\bullet EA\big(\alpha(v),\varpi(T),sh\big)dvdz.$$

From this, we could obtain the direct radiation model:

$$I(z,T,D_n)=I(0)e^{-\int_0^z\int_z^{+\infty}Ln\bullet h(v)\bullet\frac{Vb\left(\frac{v-z}{\cos(\alpha(v))}\right)}{\cos(\alpha(v))}\bullet EA\big(\alpha(v),\varpi(T),sh\big)dvdz}$$

Diffuse radiation distribution model: When the diffuse radiation is approximately isotropic in the sky and the distribution of light in the canopy is not related to the sun's location, then the expected horizontal projection area EA of a unit area leaf is:

$$EA(\alpha,T)=\frac{1}{\pi}\int_0^{\pi}\int_0^{2\pi}\tau(\beta-\varpi(T))\bullet A(\alpha,\beta,sh))d\beta dsh \quad (26)$$

and the light distribution is:

$$I(z,T,D_n)=I(0)e^{-\int_0^z\int_z^{+\infty}Ln\bullet h(v)\bullet\frac{Vb\left(\frac{v-z}{\cos(\alpha(v))}\right)}{\cos(\alpha(v))}\bullet EA\big(\alpha(v),\varpi(T)\big)dvdz} \quad (27)$$

Canopy photosynthesis rate model: By combining the canopy shape model, the canopy light distribution model and the leaf photosynthesis rate model, we got the canopy photosynthesis rate at time T:

$$PT=\int_0^{+\infty}\int_z^{+\infty}Ln\bullet h(v)\bullet\frac{Vb\left(\frac{v-z}{\cos(\alpha(v))}\right)}{\cos(\alpha(v))}\bullet p(I(z,T,D_n),N(v))dvdz \quad , \quad (28)$$

and the equation describing the canopy photosynthesis in one day is:

$$PD=\int_{T_a}^{T_b}\int_0^{+\infty}\int_z^{+\infty}Ln\bullet h(v)\bullet\frac{Vb\left(\frac{v-z}{\cos(\alpha(v))}\right)}{\cos(\alpha(v))}\bullet p(I(z,T,D_n),N(v))dvdzdT \quad (29)$$

Numerical simulation using the model: We used the canopy light distribution and photosynthesis model to investigate the effects of leaf density, leaf nitrogen content, leaf length, leaf width, leaf angle and leaf azimuth on canopy light distribution and photosynthesis through the numerical simulation of 15,625 kinds of rice plants.

Assumptions used in the simulation: The structure of the plant canopy is very complex. For our numerical simulation, we assumed some parameters, the relationships between some parameters and shape types. These assumptions are outlined in Tables 2, 3 and 4.

The numerical simulation environment: The simulation of photosynthetic rates on 15,625 plant types is very complex. We use distributed computing for the complex simulations of photosynthetic rates of 15,625 plant types. At Hunan Agricultural University, Department of Information and Computing Science Laboratory of Scientific Computing, we used 40 Lenovo PCs and a total of 80 CUPs were used to build a distributed computing environment for the simulations. These simulations took about 10 h.

Numerical experimental results: (1) As shown in the frequency histogram of the photosynthetic rates for the

15,625 plant types (Fig. 2), 15,306 plant types where the photosynthetic rates were between 40 and 120 µmol $m^{-2} s^{-1}$ and they account for 98.3% of the 15,625 plant types. There were 67 plant types where the photosynthetic rates were larger than 140 µmol $m^{-2} s^{-1}$ and they accounted for 0.43% of the 15,625 plant types. There were 252 plant types where the photosynthetic rates were less than 40 µmol $m^{-2} s^{-1}$ and they accounted for 1.61% of the 15,625 plant types. The highest photosynthetic rate plant type was [A1, A2, A3, A4, A5, A6] = [3,3,3,3,3,3], where the photosynthetic rate reached 169.21 mol $m^{-2} s^{-1}$. The lowest photosynthetic rate plant type was [A1, A2, A3, A4, A5, A6] = [3,3,3,2,2,1], where the photosynthetic rate was only 29.32 µmol $m^{-2} s^{-1}$. Based on the plant types with the highest photosynthetic rate, it can be seen that, for the ideal rice plant type, leaf density, leaf nitrogen content, leaf length, leaf width and leaf angle have large values in the upper part of the canopy and gradually decrease as you move down the plant and the azimuth is facing south. These results confirm the requirements for the super high-yielding hybrid rice plant type, which are that the length of the flag leaf is up to 50 cm and the leaf is upright (Yuan, 1999). They are also in line with Farquhar (1989) who suggests that plant photosynthesis is maximized when canopy leaf nitrogen is proportional to the radiation received by the leaf.

The canopy light intensity distribution (Fig. 3) showed that the radiation intensity on 60% of the canopy among the 78 higher photosynthetic rate plant types was over, or close to, 500 µmol $m^{-2} s^{-1}$ (close to the saturated light intensity of the leaf with highest nitrogen content) and the radiation intensity in the canopy of the 78 higher photosynthetic rate plant types decreased more slowly from the canopy top to bottom than the lower photosynthetic rate plant types (Fig. 4).

There have been very few studies on the impact of azimuth on the photosynthesis rate. As shown in Table 5, the occurrences of azimuth distribution patterns 1-5 were 14, 16, 17, 16, and 15; the frequencies were 17.95%, 20.51%, 21.79%, 20.51% and 19.23%, respectively, and the differences in photosynthetic rates were small among the plant types with the same [A1, A2, A3, A4, A5] patterns and different A6 patterns. This suggested that the effect of azimuth distribution on photosynthesis rate was very slight.

The occurrence of A5 = 3 or 5 was 74 in Table 5 and the frequency was 94.87%, the occurrence of A5 = 2 or 4 was 78 in Table 6 and the frequency was 100%, which suggested that it was a general characteristic of the superior plant types that they had more vertical leaves in the upper canopy compared with the inferior plant types.

A5=4, A1=2, A3=2 and A4=2 can also be superior plant type because the leaves in the upper canopy were sparse, short and narrow, which reduced the disadvantage of rapidly decreasing radiation as you moved down the plant. Furthermore the strong solar radiation at 12 o'clock also played an important role in reducing the disadvantage.

As shown in Table 5, A1=1, A3=1 and A4=1 did not appear, which showed that were adverse to form superior plant types. Moreover, A3 and A4 had the same values in 78 higher photosynthetic rate plant types: A3=A4=2, when A1=2; A3=A4=4, when A1=4; A3=A4= 3 or A3=A4=5, when A1=3; A3=A4= 3 or A3=A4=5, when A1=5.that is because these features can guarantee a large total LAI and enough light where the leaf area is concentrated.

As shown in Table 6, all the inferior plant types were A1,A3=1 or 3 or 5 and A4=2 or 4, that was because these plants did not have large total LAIs and there was not enough light where the leaf area was most concentrated.

As shown in Table 5, the plant types with the combination of A2 = 3 or 5 with A1, A3, A5=3 or 5 can achieve high photosynthetic rates due to strong light combined with a large leaf area and a high nitrogen content in the upper canopy. Additionally the plant type with the combination of A2 = 2, 4 with A1, A3, A5=2, 4 can also achieve high photosynthetic rates due to a large leaf area with high nitrogen levels in the middle canopy, strong solar radiation at 12 o'clock and sparse, short and narrow leaves in the upper canopy, which allowed enough strong light to enter into the middle canopy. That led us to the conclusion that the connection between strong light, large leaf area and high nitrogen content is crucial to the superior plant types.

Table 2. Assumptions used for certain parameters.

Nitrogen content	0.007-0.035 g kg^{-1}	Range of leaf length	20 cm -50cm
Range of leaf width	0.6cm-2.8cm	Rang of leaf angle	35degree-88degree
Range of leaf base position	2cm-100cm	I_0E_0	1500µmol$m^{-2}s^{-1}$
Altitude	28 degree12 cent	The order of day	180
Time	12 o'clock		

Table 3. Relationship between parameters used to measure photosynthesis and nitrogen content.

The curvature factor	$\phi(N) = 0.055 + 0.025*(N - 0.7\%)/(3.5\% 0.7\%)$
the initial slope	$\theta = 0.85$
the light-saturated photosynthetic rate	P_{max} = 6+24*(*N*-0.007)/0.028

Table 4. Patterns for leaf number density, nitrogen content, length, width, angle and azimuth.

	Pattern 1	Pattern 2	Pattern 3	Pattern 4	Pattern 5
Leaf density (A1)	M1(z)	M2(z)	M3(z)	M4(z)	M5(z)
Nitrogen content (A2)	M6(0.007, 0.035,z)	M7(0.007, 0.035,z)	M8(0.007, 0.035,z)	M9(0.007, 0.035,z)	M10(0.007, 0.035,z)
Leaf length (A3)	M6(20,50,z)	M7(20,50,z)	M8(20,50,z)	M9(20,50,z)	M10(20,50,z)
Leaf width (A4)	M6(0.6,2.8,z)	M7(0.6,2.8,z)	M8(0.6,2.8,z)	M9(0.6,2.8,z)	M10(0.6,2.8,z)
Leaf angle (A5)	M6(35,88,z)	M7(35,88,z)	M8(35,88,z)	M9(35,88,z)	M10(35,88,z)
Leaf azimuth (A6)	M11(beta)	M12(beta)	M13(beta)	M14(beta)	M15(beta)

Note: In Table 4, the six factors have five modes, respectively, and the total is 15625, which equals 56 plant types; The five modes illustrate the leaf density function, nitrogen content function, length function, width function, angle function and the azimuth function of the leaf base position. The corresponding pseudo-code is:

$M_1(z) = \frac{1}{z_{max} - z_{min}}$; $M_2(z) = \frac{2z}{z_{max}^2 - z_{min}^2}$; $M_3(z) = \frac{2(z_{max} - z)}{z_{max}^2 - z_{min}^2}$; $M_4 = \frac{2(z_{max} - |z - (z_{max} + z_{min})/2|)}{z_{max}^2 - z_{min}^2}$

$M_5 = \frac{2(z_{min} + |z - (z_{max} + z_{min})/2|)}{z_{max}^2 - z_{min}^2}$ $M_6(\min, \max, z) = \frac{\min + \max}{2}$

$M_7(\min, \max, z) = \min + (\max - \min)\frac{z - zmin}{zmax - zmin}$ $M_8(\min, \max, z) = \min + (\max - \min)\frac{zmax - z}{zmax - zmin}$

$$M_9(\min, \max, z) = \max - (\max - \min)\frac{2|z - (zmax - zmin)/2|}{zmax + zmin}$$

$M_{10}(\min, \max, z) = \min + (\max - \min)\frac{2|z - (zmax - zmin)/2|}{zmax + zmin}$ $M_{11}(\beta) = \frac{1}{2\pi}$

$M_{12}(\beta) = \begin{cases} \frac{1}{2.2\pi} & \beta \in (0.5\pi, 1.5\pi) \\ \frac{q_1}{2.2\pi} & \beta \in [0, 2\pi]/(0.5\pi, 1.5\pi) \end{cases}$ $M_{13}(\beta) = \begin{cases} \frac{1}{2.2\pi} & \beta \in (\pi, 2\pi] \\ \frac{q_2}{2.2\pi} & \beta \in [0, \pi) \end{cases}$

$M_{12}(\beta) = \begin{cases} \frac{1}{2.2\pi} & \beta \in [0, 2\pi]/(0.5\pi, 1.5\pi) \\ \frac{q_1}{2.2\pi} & \beta \in (0.5\pi, 1.5\pi) \end{cases}$ $M_{12}(\beta) = \begin{cases} \frac{1}{2.2\pi} & \beta \in [0, \pi) \\ \frac{q_2}{2.2\pi} & \beta \in (\pi, 2\pi] \end{cases}$

where, $q_1 = \sqrt{(\tau_{max} * \cos(\beta))^2 + (\tau_{min} * \sin(\beta))^2}$; $q_2 = \sqrt{(\tau_{max} * \sin(\beta))^2 + (\tau_{min} * \cos(\beta))^2}$;

$z_{max} = 100, z_{min} = 2, z_{min} < z < z_{max}, \tau_{min} = 1, \tau_{max} = 1.2$

Conclusion and Discussion

Conclusion of the numerical experiment: Based on the above analysis we formed the following conclusions: (1) The ideal rice plant type has the following characteristics: the values for leaf density, leaf length, leaf width, leaf angle and leaf nitrogen content are largest at the upper part of canopy and gradually declined as you moved down the plant. (2) The radiation intensity decreased more slowly in the canopy of the better plant types, whereas the radiation intensity decreased more rapidly with the less adapted plant types. (3) The better plant types generally had a larger leaf angle in the upper canopy. In contrast the less well adapted plant types commonly had a small leaf angle in the upper canopy. (4) It is crucial for the better plant types that they have a large leaf area, high nitrogen levels and enough solar radiation. (5) The affect of azimuth distribution on photosynthesis rate is very slight. However, the plant type with south facing leaves had a better photosynthetic rate. South facing leaves may be the result of natural selection for improved photosynthetic rates. (6) The connection between strong light, large leaf area and high nitrogen content is an important factor when selecting improved plant types.

Model characteristics and their significance: (1) The measurement of the leaf area and the extinction coefficient at every layer is needed if the photosynthetic rate is to be computed by currently available methods. However, in our model, what is needed for computing rice photosynthetic rate is the measurement of leaf base position, leaf angle, leaf azimuth, leaf length, leaf width and leaf nitrogen content through random sampling.

When using our model to measure photosynthetic rates, you do not need to measure the layered leaf area. This avoids cutting the canopy, which is destructive to the canopy and does not allow continued study of the whole process of plant growth.

Table 5. The data for 78 plant types that had a higher photosynthesis rate. P: photosynthesis rate (µmol m^{-2} s^{-1}).

No	A1	A2	A3	A4	A5	A6	LAI	P	No	A1	A2	A3	A4	A5	A6	LAI	P
1.	3	3	3	3	3	3	7.56	169.2	40.	3	1	3	3	3	3	6.7	144.5
2.	3	3	3	3	3	4	7.56	169.2	41.	3	1	3	3	3	4	6.7	144.4
3.	3	3	3	3	3	2	7.56	169.2	42.	3	1	3	3	3	2	6.7	144.4
4.	3	3	3	3	3	5	7.56	169.1	43.	3	1	3	3	3	5	6.7	144.3
5.	3	3	3	3	3	1	7.56	168.9	44.	3	1	3	3	3	1	6.7	144.2
6.	4	4	4	4	3	3	8.02	168.3	45.	2	2	2	2	4	3	8.23	143.2
7.	4	4	4	4	3	4	8.02	167.7	46.	5	5	5	5	3	1	7.56	142.9
8.	4	4	4	4	3	2	8.02	167.7	47.	2	2	2	2	4	4	8.23	142.5
9.	4	4	4	4	3	5	8.02	167.2	48.	2	2	2	2	4	2	8.23	142.5
10.	3	3	3	3	5	3	7.56	166.4	49.	3	3	5	3	3	3	6.51	141.6
11.	3	3	3	3	5	4	7.56	166.4	50.	3	3	5	3	3	4	6.51	141.6
12.	3	3	3	3	5	2	7.56	166.4	51.	3	3	5	3	3	2	6.51	141.6
13.	3	3	3	3	5	5	7.56	166.2	52.	2	2	2	2	4	5	8.23	141.6
14.	3	3	3	3	5	1	7.56	165.5	53.	3	3	5	3	3	5	6.51	141.6
15.	4	4	4	4	3	1	8.02	164.4	54.	3	3	5	3	3	1	6.51	141.5
16.	3	5	3	3	3	3	6.72	148.9	55.	3	1	3	3	5	3	6.7	141.4
17.	3	5	3	3	3	4	6.72	148.8	56.	3	1	3	3	5	4	6.7	141.3
18.	3	5	3	3	3	2	6.72	148.8	57.	3	1	3	3	5	2	6.7	141.3
19.	3	5	3	3	3	5	6.72	148.8	58.	3	1	3	3	5	5	6.7	141.1
20.	3	5	3	3	3	1	6.72	148.6	59.	3	3	5	3	5	3	6.51	140.7
21.	3	5	3	3	5	3	6.72	147	60.	3	3	5	3	5	4	6.51	140.7
22.	3	5	3	3	5	4	6.72	147	61.	3	3	5	3	5	2	6.51	140.7
23.	3	5	3	3	5	2	6.72	147	62.	3	3	5	3	5	5	6.51	140.6
24.	3	5	3	3	5	5	6.72	146.8	63.	3	1	3	3	5	1	6.7	140.3
25.	5	3	3	3	3	3	6.64	146.8	64.	3	3	5	3	5	1	6.51	140.3
26.	5	3	3	3	3	4	6.64	146.8	65.	3	3	3	5	3	3	7.56	140
27.	5	3	3	3	3	2	6.64	146.8	66.	3	3	3	5	3	4	7.56	140
28.	5	5	5	5	3	3	7.56	146.8	67.	3	3	3	5	3	2	7.56	140
29.	5	3	3	3	3	5	6.64	146.8	68.	3	3	3	5	3	5	7.56	139.9
30.	5	3	3	3	3	1	6.64	146.6	69.	3	4	3	3	3	3	6.67	139.7
31.	5	5	5	5	3	4	7.56	146.5	70.	3	4	3	3	3	2	6.67	139.7
32.	5	5	5	5	3	2	7.56	146.5	71.	3	4	3	3	3	4	6.67	139.7
33.	3	5	3	3	5	1	6.72	146.4	72.	3	3	3	5	3	1	7.56	139.7
34.	5	5	5	5	3	5	7.56	146	73.	3	4	3	3	3	5	6.67	139.6
35.	5	3	3	3	5	3	6.64	145.4	74.	3	4	3	3	3	1	6.67	139.4
36.	5	3	3	3	5	4	6.64	145.4	75.	5	5	5	5	5	3	7.56	138.3
37.	5	3	3	3	5	2	6.64	145.4	76.	5	5	5	5	5	4	7.56	138
38.	5	3	3	3	5	5	6.64	145.3	77.	5	5	5	5	5	2	7.56	138
39.	5	3	3	3	5	1	6.64	145	78.	4	2	4	4	3	3	6.86	137.9

Table 6. Data for 78 plant types that had lower photosynthesis rates (μmol m-^{2} s^{-1}).

No	A1	A2	A3	A4	A5	A6	LAI	P	No	A1	A2	A3	A4	A5	A6	LAI	P
1.	3	3	3	2	2	1	7.8	29.3	40.	3	1	3	2	4	1	6.9	33.1
2.	3	5	3	2	2	1	6.9	29.5	41.	5	5	3	2	2	5	6.5	33.2
3.	5	3	3	2	2	1	6.7	29.7	42.	3	5	5	2	2	5	6.5	33.3
4.	3	3	5	2	2	1	6.7	29.7	43.	5	5	3	2	2	2	6.5	33.3
5.	3	3	3	2	4	1	7.8	30.2	44.	5	5	3	2	2	4	6.5	33.3
6.	5	5	3	2	2	1	6.5	30.5	45.	5	3	3	2	4	5	6.7	33.4
7.	3	5	5	2	2	1	6.5	30.5	46.	3	3	5	2	4	5	6.7	33.4
8.	5	3	3	2	4	1	6.7	30.7	47.	3	5	5	2	2	2	6.5	33.4
9.	3	3	5	2	4	1	6.7	30.7	48.	3	5	5	2	2	4	6.5	33.4
10.	3	5	3	2	4	1	6.9	30.8	49.	5	3	3	2	4	2	6.7	33.5
11.	3	1	3	2	2	1	6.9	31.3	50.	5	3	3	2	4	4	6.7	33.5
12.	5	3	5	2	2	1	6.8	31.6	51.	5	5	3	2	2	3	6.5	33.5
13.	3	3	3	2	2	5	7.8	31.8	52.	3	3	5	2	4	2	6.7	33.5
14.	3	3	3	2	2	2	7.8	31.9	53.	3	3	5	2	4	4	6.7	33.5
15.	3	3	3	2	2	4	7.8	31.9	54.	3	5	5	2	2	3	6.5	33.5
16.	3	5	3	2	2	5	6.9	32	55.	3	5	3	2	4	5	6.9	33.6
17.	3	5	3	2	2	2	6.9	32.1	56.	5	3	3	2	4	3	6.7	33.6
18.	3	5	3	2	2	4	6.9	32.1	57.	3	3	5	2	4	3	6.7	33.6
19.	3	3	3	2	2	3	7.8	32.1	58.	3	5	3	2	4	2	6.9	33.7
20.	3	5	3	2	2	3	6.9	32.2	59.	3	5	3	2	4	4	6.9	33.7
21.	5	3	3	2	2	5	6.7	32.2	60.	3	5	3	2	4	3	6.9	33.8
22.	3	1	5	2	2	1	6	32.2	61.	1	5	3	2	2	1	6	34.1
23.	3	3	5	2	2	5	6.7	32.2	62.	3	1	3	2	2	5	6.9	34.1
24.	5	3	3	2	2	2	6.7	32.3	63.	3	1	3	2	2	2	6.9	34.2
25.	5	3	3	2	2	4	6.7	32.3	64.	3	1	3	2	2	4	6.9	34.2
26.	3	3	5	2	2	2	6.7	32.3	65.	3	5	1	2	2	1	6	34.3
27.	3	3	5	2	2	4	6.7	32.3	66.	3	1	3	2	2	3	6.9	34.3
28.	5	3	3	2	2	3	6.7	32.4	67.	3	4	3	2	2	1	6.8	34.3
29.	3	3	5	2	2	3	6.7	32.4	68.	3	5	3	4	2	1	6.9	34.4
30.	5	5	3	2	4	1	6.5	32.4	69.	5	5	5	2	2	1	7.8	34.5
31.	5	1	3	2	2	1	6	32.5	70.	3	1	5	2	4	1	6	34.6
32.	3	5	5	2	4	1	6.5	32.5	71.	5	3	3	4	2	1	6.7	34.6
33.	5	3	5	2	4	1	6.8	32.9	72.	3	3	5	4	2	1	6.7	34.7
34.	1	3	3	2	2	1	6.5	32.9	73.	3	3	3	4	2	1	7.8	34.7
35.	3	3	3	2	4	5	7.8	32.9	74.	5	3	5	2	2	5	6.8	34.7
36.	3	3	1	2	2	1	6.5	33	75.	3	4	5	2	2	1	5.4	34.7
37.	3	3	3	2	4	2	7.8	33	76.	5	3	5	2	2	2	6.8	34.8
38.	3	3	3	2	4	4	7.8	33	77.	5	3	5	2	2	4	6.8	34.8
39.	3	3	3	2	4	3	7.8	33.1	78.	5	3	5	2	2	3	6.8	34.9

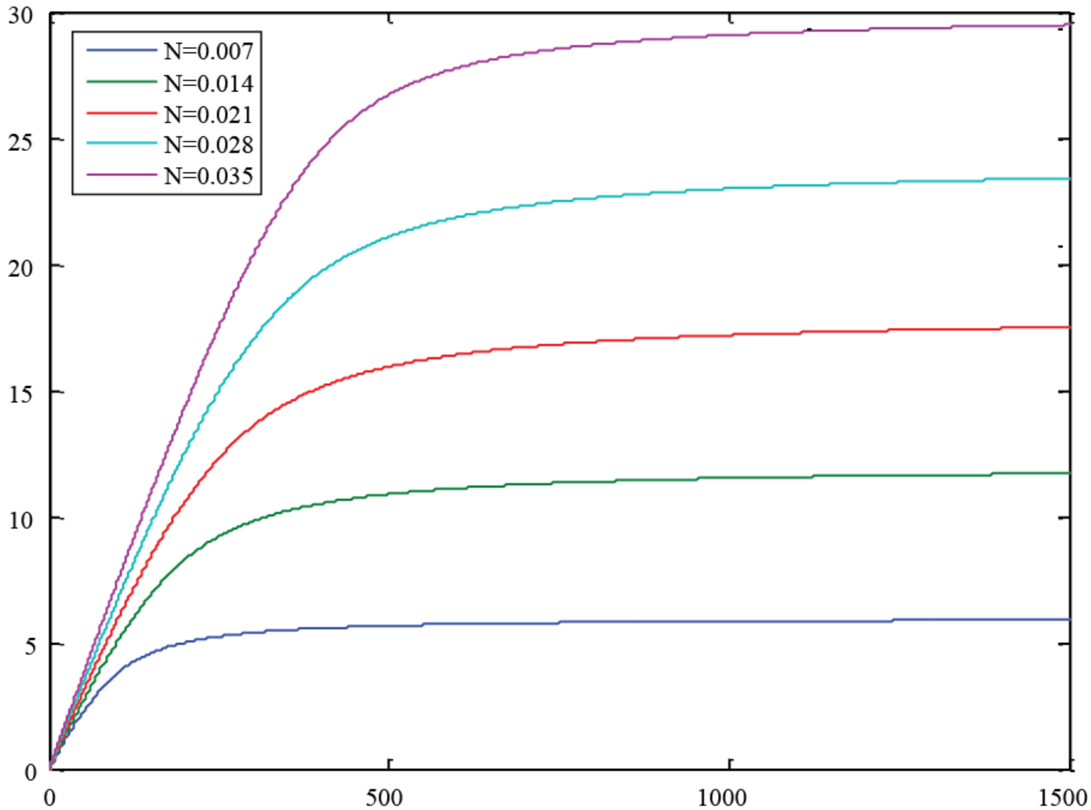

Fig. 1. Photosynthetic rate responses to light intensity at different nitrogen levels.

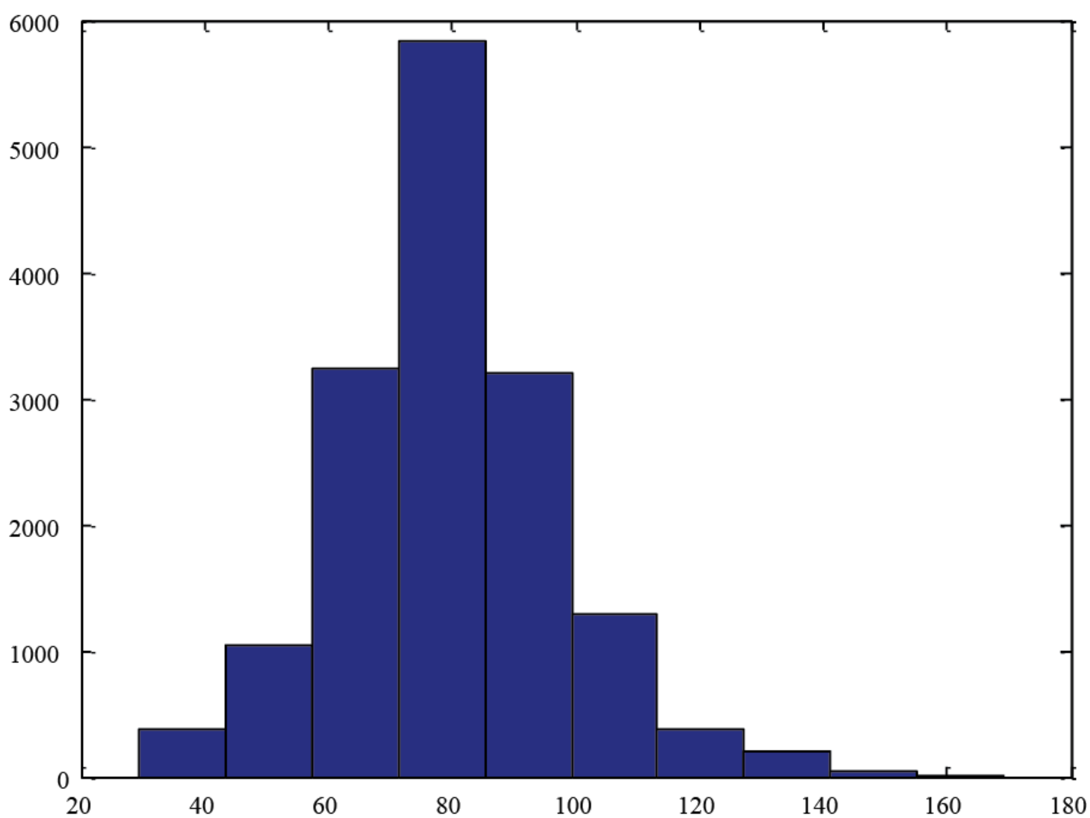

Fig. 2. Distribution histogram for the photosynthetic rates.

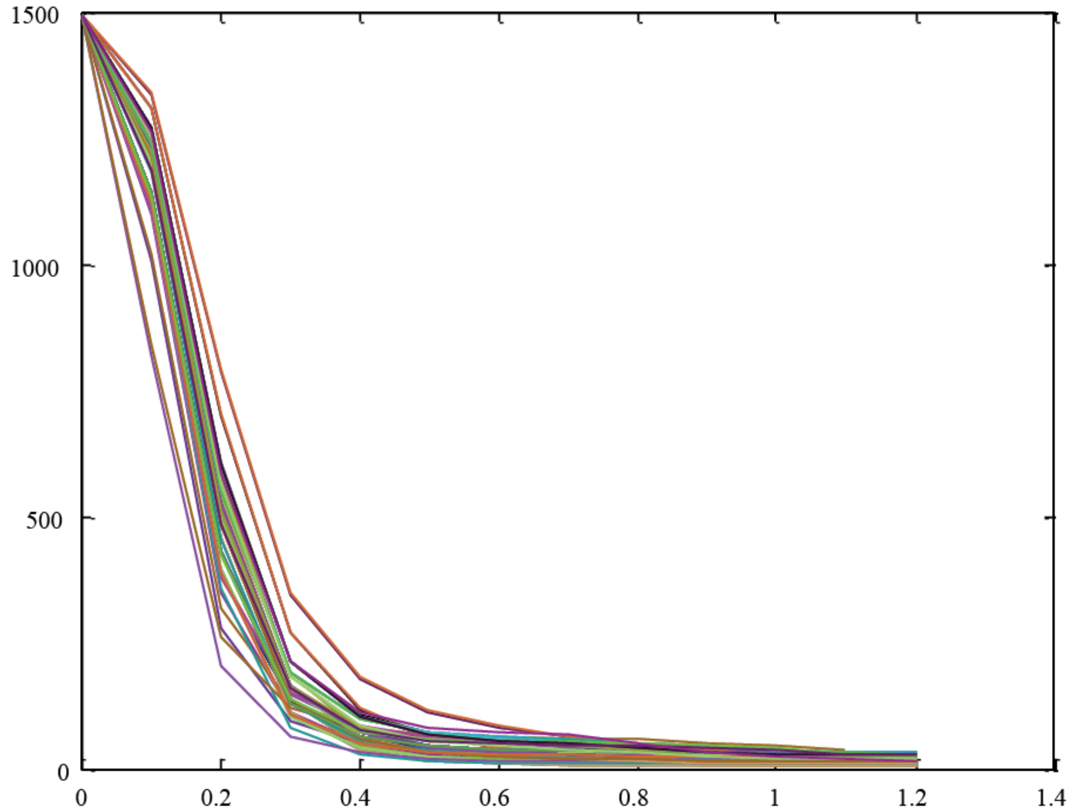

Fig. 3. The light distribution in the canopy of 78 plant types with higher photosynthesis rates.

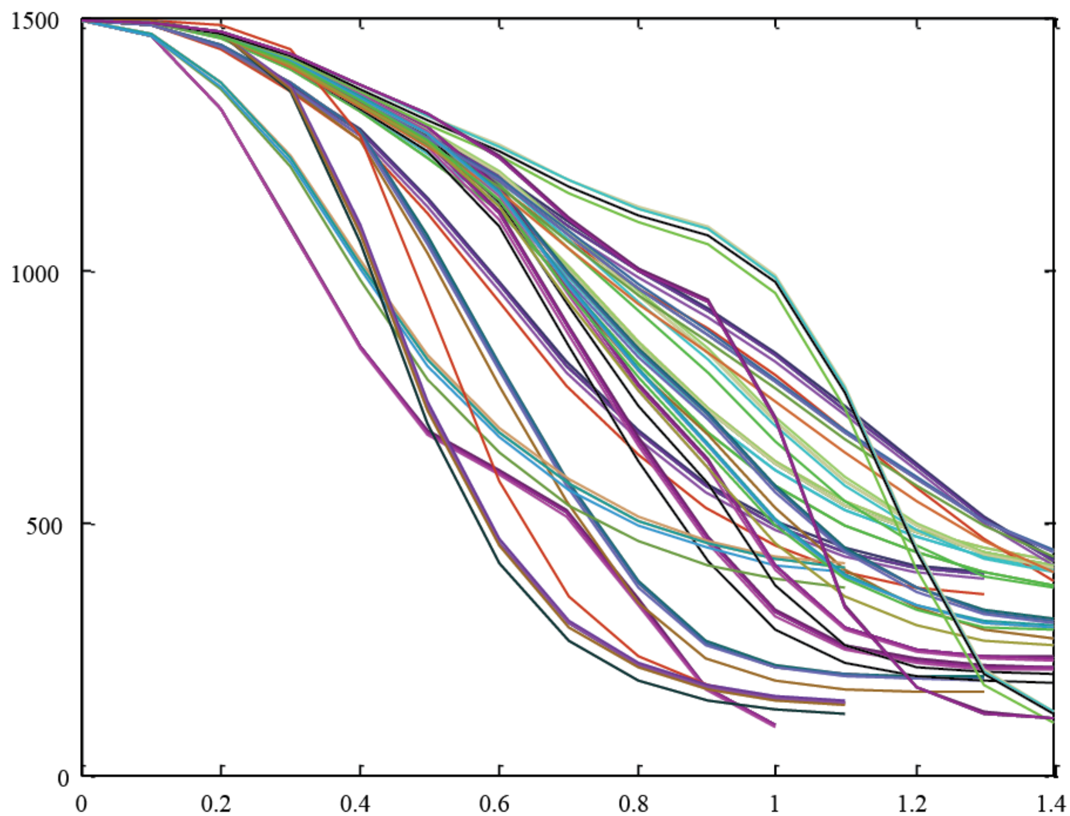

Fig. 4. The light distribution in the canopy of 78 plant types with lower photosynthesis rates.

The factors, which vary along the stem, such as leaf density, leaf length, leaf width, leaf angle, leaf azimuth and leaf nitrogen content, are all considered in our photosynthesis model. Other important factors, such as temperature, can also be included in our photosynthesis model. This advantage should allow the development of a new mathematical model that could be used to study canopy photosynthesis.

Our model proposes a new method and idea for evaluating and optimizing plant types. Moreover, it can be used to improve plant computer simulation technology.

Improvement and development of the model: New model improvements often have to go through a number of stages. Our model is no exception. Firstly, many assumptions in our model need further examination and correction. Secondly, many details in this model need further improvement, for example, respiration is not a parameter in this model

Based on our model, the following aspects should be investigated further:(1) Assumptions testing. Test, revise and improve the assumptions in the model through laboratory experiments or field trials. (2) Model extension. More factors that are important to cereal photosynthesis and may vary in space should be included in the model. (3) Model Application. Our model, and other new models based on our method, should be independently evaluated and the predicted optimal of plant type should be confirmed in practice. New crop growth simulation software should be developed in combination with computer simulation technology.

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